Product differentiation

How far does a market extend?
Which firms compete with each other?
What is an industry?

Products are *not* homogeneous.
Exceptions: petrol, electricity.

But some products are more equal to each other than to other products in the economy. These products constitute an industry.

A market with *product differentiation*.

But: where do we draw the line?

Example:

- beer vs. soda?
- soda vs. milk?

- beer vs. milk?

Two kinds of product differentiation

(i) Horizontal differentiation: Consumers differ in their preferences over the product's characteristics. Examples: colour, taste, location of outlet.

(ii) Vertical differentiation: Products differ in some characteristic in which all consumers agree what is best. Call this characteristic quality. (*quality competition*)

Horizontal differentiation

Two questions:

1. Is the product variation too large in equilibrium?

2. Are there too many variants in equilibrium?

Question 1: A fixed number of firms. Which product variants will they choose?

Question 2: Variation is maximal. How many firms will enter the market?

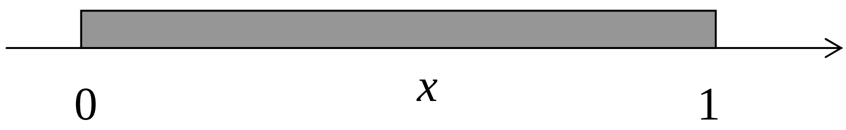
The two questions call for different models.

Variation in equilibrium

Will products supplied in an unregulated market be too similar or too different, relative to social optimum?

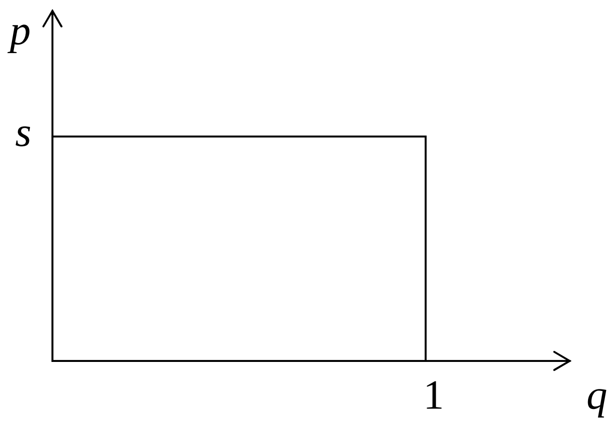
Hotelling (1929)

Product space: the line segment [0, 1].
Two firms: one at 0, one at 1.



Consumers are uniformly distributed along [0, 1].
A consumer at x prefers the product variety x.

Consumers have unit demand:



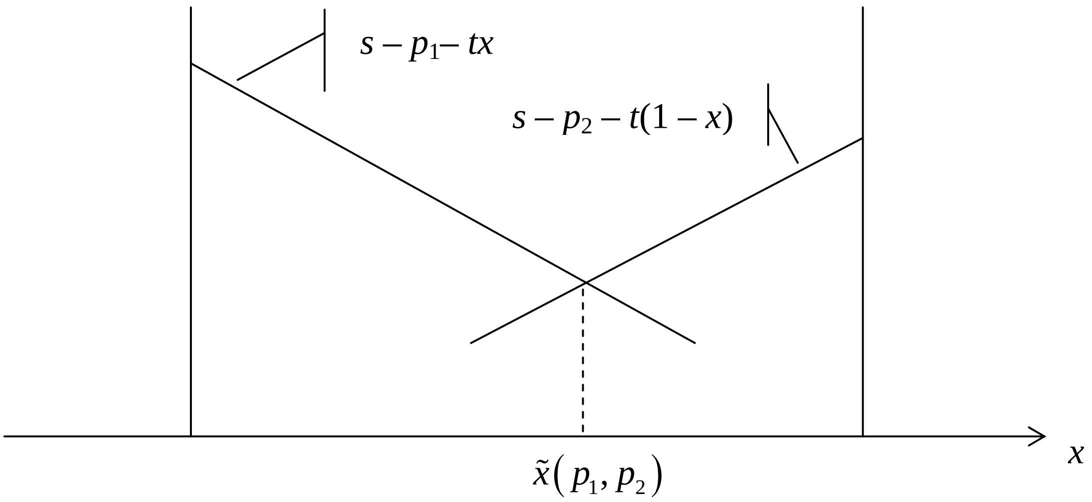
Disutility from consuming product variety y:

$t(|y - x|)$ – ‘‘transportation costs”

Linear transportation costs: $t(d) = td$

Generalised prices (with firm 1 at 0 and firm 2 at 1):

$p_1 + tx$ and $p_2 + t(1 - x)$



The indifferent consumer: $\tilde{x}$

$$s - p_1 - t\tilde{x} = s - p_2 - t(1 - \tilde{x}).$$

$$\Rightarrow \tilde{x}(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

[But check that: (i) $0 \leq \tilde{x} \leq 1$; (ii) $\tilde{x}$ wants to buy.]

Normalizing the number of consumers: $N = 1$ (thousand)

$$D_1(p_1, p_2) = \tilde{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

$$D_2(p_1, p_2) = 1 - \tilde{x} = \frac{1}{2} + \frac{p_1 - p_2}{2t}$$

Constant unit cost of production: c

$$\pi_1(p_1, p_2) = (p_1 - c)\left[\frac{1}{2} + \frac{p_2 - p_1}{2t}\right]$$

Price competition.

Equilibrium conditions: $\frac{\partial \pi_1}{\partial p_1} = 0;\ \frac{\partial \pi_2}{\partial p_2} = 0$

FOC[1]:

$$\underbrace{(p_1 - c)\left(-\frac{1}{2t}\right)}_{\text{increased price reduces sales}} + \underbrace{\frac{1}{2} + \frac{p_2 - p_1}{2t}}_{\text{increased price increases gain per unit sold}} = 0$$

$\Rightarrow$ FOC[1]: $2p_1 - p_2 = c + t$

FOC[2]: $2p_2 - p_1 = c + t$

$\Rightarrow$ $p_1^* = p_2^* = c + t$

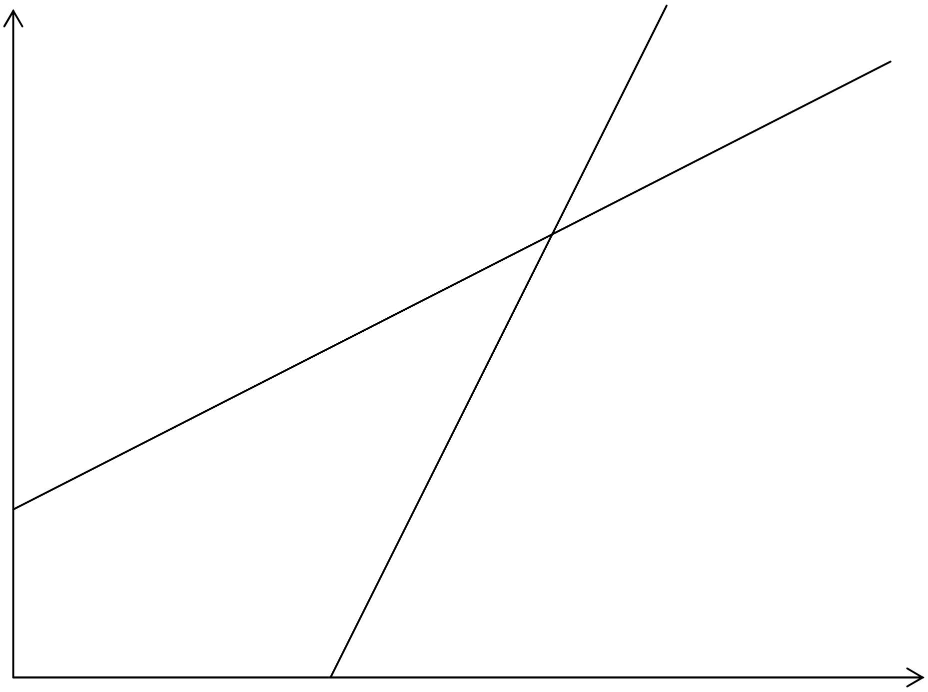
- The indifferent consumer does want to buy if:

$$s \geq c + \tfrac{3}{2}t$$

- Prices are *strategic complements*:

$$\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} = \frac{1}{2t} > 0$$

Best-response function: $p_1 = ½(p_2 + c + t)$



The degree of product differentiation: t

Product differentiation makes firms less aggressive in their pricing.

But are 0 and 1 the firms' equilibrium product variations?

Two-stage game of product differentiation:

Stage 1: Firms choose locations on [0, 1].

Stage 2: Firms choose prices.

Linear vs. convex transportation costs.

- Convex costs analytically tractable but economically less meaningful?

Assume quadratic transportation costs.

Stage 2:
Firms 1 and 2 located at a and $1 - b$, $a \geq 0$, $b \geq 0$, $a + b \leq 1$.

The indifferent consumer:

$$p_1 + t(\tilde{x} - a)^2 = p_2 + t(1 - b - \tilde{x})^2$$

$$\tilde{x} = a + \frac{1}{2}(1 - a - b) + \frac{p_2 - p_1}{2t(1 - a - b)}$$

$$D_1(p_1, p_2) = \tilde{x}, \quad D_2(p_1, p_2) = 1 - \tilde{x}$$

$$\pi_1(p_1, p_2) = (p_1 - c)\left[a + \frac{1}{2}(1 - a - b) + \frac{p_2 - p_1}{2t(1 - a - b)}\right]$$

Equilibrium conditions: $\frac{\partial \pi_1}{\partial p_1} = 0;\ \frac{\partial \pi_2}{\partial p_2} = 0$

FOC[1]: $2p_1 - p_2 = c + t(1 - a - b)(1 + a - b)$

FOC[2]: $2p_2 - p_1 = c + t(1 - a - b)(1 - a + b)$

Equilibrium:

$$p_1 = c + t(1 - a - b)\left(1 + \frac{a - b}{3}\right)$$
$$p_2 = c + t(1 - a - b)\left(1 + \frac{b - a}{3}\right)$$

- Symmetric location: $a = b \Rightarrow p_1 = p_2 = c + t(1 - 2a)$
- A firm's price decreases when the other firm gets closer: $\frac{dp_1}{db} < 0$.
- Stage-2 outcome depends on locations:
  $p_1 = p_1(a, b),\ \ p_2 = p_2(a, b)$

Stage 1:

$\pi_1(a, b) = [p_1(a, b) - c]D_1(a, b, p_1(a, b), p_2(a, b))$

$$\frac{d\pi_1}{da} = D_1 \frac{\partial p_1}{\partial a} + (p_1 - c)\left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_1}\frac{\partial p_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a}\right]$$

$$= \underbrace{\left[D_1 + (p_1 - c)\frac{\partial D_1}{\partial p_1}\right]}_{=0}\frac{\partial p_1}{\partial a} + (p_1 - c)\left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a}\right]$$

$$\frac{d\pi_1}{da} = (p_1 - c)(\underbrace{\frac{\partial D_1}{\partial a}}_{\substack{\text{direct}\\ \text{effect;}\\ >0}} + \underbrace{\overbrace{\frac{\partial D_1}{\partial p_2}}^{>0}\overbrace{\frac{\partial p_2}{\partial a}}^{<0}}_{\substack{\text{strategic}\\ \text{effect;}\\ <0}})$$

Moving toward the middle:
A positive direct effect vs. a negative strategic effect.

$$\frac{\partial D_1}{\partial a} = \frac{1}{2} + \frac{p_2 - p_1}{2t(1-a-b)^2} = \frac{1}{2} + \frac{b-a}{3(1-a-b)}$$

$$= \frac{3-5a-b}{6(1-a-b)} > 0, \text{if } a \leq \frac{1}{2}$$

$$\frac{\partial p_2}{\partial a} = \frac{2}{3}t(a-2) < 0$$

$$\frac{\partial D_1}{\partial p_2} = \frac{1}{2t(1-a-b)} > 0$$

$$\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a} = \frac{3-5a-b}{6(1-a-b)} + \frac{a-2}{3(1-a-b)} = -\frac{3a+b+1}{6(1-a-b)} < 0$$

Equilibrium: $a^* = b^* = 0$.

Strategic effect stronger than direct effect.
Maximum differentiation in equilibrium.

Social optimum:

No quantity effect. Social planner wants to minimize total transportation costs. (Kaldor-Hicks vs. Pareto)

In social optimum, the two firms split the market and locate in the middle of each segment: ¼ and ¾.

In equilibrium, product variants are too different.

- Crucial assumption: convex transportation costs.

- Also other equilibria, but they are in mixed strategies.
  [Bester *et al.*, ‘‘A Noncooperative Analysis of Hotelling’s Location Game”, *Games and Economic Behavior* 1996]

- Multiple dimensions of variations: Hotelling was almost right
  [Irmen and Thisse, ”Competition in multi-characteristics spaces: Hotelling was almost right”, *Journal of Economic Theory* 1998]

- Head-to-head competition in shopping malls: Consumers poorly informed?
  [Klemperer, “Equilibrium Product Lines”, *AER* 1992]

Have we really solved the problem whether or not the equilibrium provision of product variants has too much or too little differentiation?